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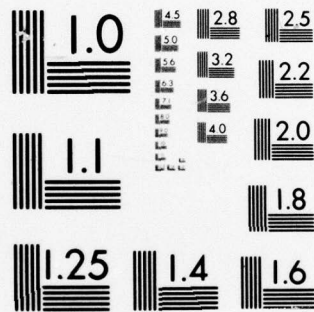


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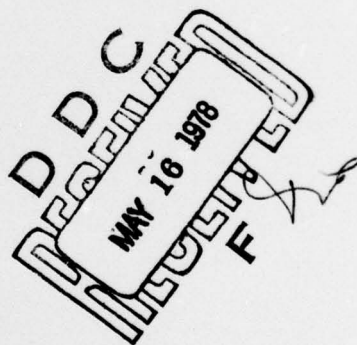
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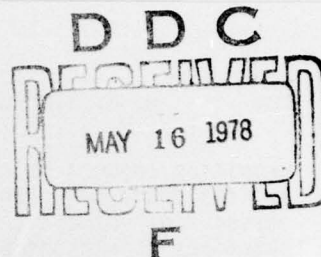
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# PROGRESS ON A BASE FLOW MODEL FOR EXTERNAL BURNING PROPULSION

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## Abstract

Progress is described on a model of the turbulent axisymmetric base flow behind bluff-base bodies. The purpose of the model is to provide an analytical basis for incorporation of external burning and base injection effects upon the base flow. An integral technique is used for solution of the problem. At the current time, however, extreme difficulties are being encountered in the choice of field variable profiles to insert in the integral technique. Several choices of profiles and governing moment equations are yielding a singularity before the rear stagnation point is reached in a downstream integration of the equation.

## Introduction

This is an extension of the treatment of Ref. 1 to treat the axisymmetric case with a more realistic base flow and combustion theory. This extension is now warranted because of the experimental proof, obtained elsewhere, that the concept of external burning behind bluff base bodies will indeed work in a propulsion mode.

It is first necessary to construct an axisymmetric, turbulent base flow theory without reaction which contains sufficient detail to eventually incorporate the many phenomena which will be of interest in the external burning case. First, the reversed flow region must be treated with enough detail to include estimates of the reversed flow velocity distribution and to allow for base bleed. The reactive bleed may be a

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promising adjunct to external burning to improve performance and residence times in the near wake are influenced by the reversed flow velocity. Second, several forms of adiabatic assumptions should not be used because reactive matter will be entrained in the near wake, and may be introduced by bleed. Consequently, the energy equation must be used in some form. Furthermore, an entropy layer in the inviscid stream must be allowed for, because several concepts of external burning would introduce the fuel by injection into a supersonic stream, causing injection shocks. The approach outlined below is ultimately capable of treating these phenomena.

#### Analysis

The approach followed is similar to that of Alber<sup>2</sup> but allows for the foregoing complexities and the axisymmetric geometry. The equations being used are the usual approximate boundary layer equations for turbulent flow of the near wake with a turbulent Prandtl number of unity and assuming a perfect gas. These are

$$\partial/\partial x (\rho u) + 1/r \partial/\partial r (\rho u r) = 0 \quad (1)$$

$$\rho u \partial u/\partial x + \rho v \partial u/\partial r = - dp/dx + 1/r \partial/\partial r (r \tau_T) \quad (2)$$

$$\rho u \partial H/\partial x + \rho v \partial H/\partial r = - 1/r \partial/\partial r (r \mu_T \partial H/\partial r) \quad (3)$$

$$\tau_T = \mu_T \partial u/\partial r \quad (4)$$

$$H = h + u^2/2 \quad (5)$$

$$h = c_p T \quad p = \rho R T \quad (6)$$

In Eqs. (1-6)  $\rho$  is the density,  $u$  is the axial velocity in the  $x$ -direction,  $v$  is the radial velocity in the  $r$ -direction,  $p$  is the pressure,  $\tau_T$  is the turbulent shear stress,  $\mu_T$  is the turbulent "viscosity,"  $H$  is the stagnation enthalpy,  $h$  is the static enthalpy,  $c_p$  is the specific heat at constant pressure,  $T$  is the temperature, and  $R$  is the gas constant.

An integral technique, a variant of the method of weighted residuals,<sup>3</sup> is used to solve the system of equations. If we integrate Eq. (2) from 0 to  $\delta$ , where  $\delta$  is the near wake thickness, and use Eq. (1), the familiar momentum equation is obtained:

$$d/dx (\rho_e u_e^2 A_2) + \rho_e u_e A_1 du_e/dx = 0$$



$$A_1 = \int_0^{\delta} (1 - \rho u / \rho_e u_e) r dr, \quad A_2 = \int_0^{\delta} \rho u / \rho_e u_e (1 - u / u_e) r dr \quad (7)$$

Here  $A_1$  is the displacement "area" and  $A_2$  is the momentum area. Multiplying Eq. (2) by  $u$  and integrating across the layer, the equation for the first velocity moment of momentum becomes the following:

$$d/dx (\rho_e u_e^3 A_3) + 2 \rho_e u_e^2 (A_1 - A_u) du_e/dx = 2 \int_0^{\delta} \mu_T (\partial u / \partial r)^2 r dr$$

$$A_3 = \int_0^{\delta} \rho u / \rho_e u_e (1 - u^2 / u_e^2) r dr, \quad A_u = \int_0^{\delta} (1 - u / u_e) r dr \quad (8)$$

Integrating the energy equation, Eq. (3), and using Eq. (5),

$$d/dx (\rho_e u_e h_e A_H) + 1/2 d/dx (\rho_e u_e^3 A_3) + dH_e/dx \int_0^{\delta} \rho u r dr = 0$$

$$A_H = \int_0^{\delta} \rho u / \rho_e u_e (h / h_e - 1) r dr \quad (9)$$

where  $H_e$  is the stagnation enthalpy at the edge of the near wake and  $dH_e/dx = 0$  for the present developments. Integrating Eq. (1) over the layer

$$\rho_e v_e \delta - d/dx (\rho_e u_e A_1) + \delta^2 / 2 d/dx (\rho_e u_e) = 0 \quad (10)$$

Since  $dP_e/dx = dp/dx$  and  $-\rho_e u_e du_e/dx$  under the approximations of this analysis and the external flow is isentropic,  $\rho_e = \rho_e(u_e)$  and  $h_e = h_e(u_e)$ .

In order to use Eqs. (7-10), profiles of velocity and enthalpy must be chosen. The profiles are chosen to yield algebraic simplicity while containing enough of the essential physics of the problem. To start the integration from the base, the profiles of Green<sup>4</sup> are used, following Alber. These are

$$\begin{aligned} \rho u / \rho_e u_e &= k - 2P & 0 \leq r \leq h \\ &= 1 - P - P \cos \pi(r-h)/\ell & h \leq r \leq \delta \end{aligned}$$

$$\begin{aligned}
 h/h_e &= 1 - 2P_H & 0 \leq r \leq h \\
 &= 1 - P_H - P_H \cos \pi(r-h)/\ell & h \leq r \leq \delta \quad (11)
 \end{aligned}$$

Here  $\ell(x)$  is the thickness of the shear layer and  $h(x)$  is the height of the roughly constant velocity region of reversed flow.  $\delta = h + \ell$  and  $P(x)$  and  $P_H(x)$  are profile parameters.

The unknowns in Eqs. (7-11) are  $\ell$ ,  $h$ ,  $P$ ,  $P_H$ ,  $u_e$ , and  $v_e$ , whereas there are only four differential equations in Eqs. (7-10). Consequently, two more pieces of information are required. One is obviously information about the external flow. This is treated by Webb's approximate method of characteristics,<sup>5</sup> which essentially ignores left running characteristic lines and allows a point-by-point integration in  $x$  without carrying a full characteristics set downstream. This method allows computation of the exterior flow angle and the flow velocity vector, which yields  $v_e = v_e(u_e)$ . The remaining requirement is an additional differential equation. Several options are open: 1) various forms of centerline equations as used by Alber<sup>2</sup> and Ohrenberger and Baum,<sup>6</sup> 2) additional velocity moments of the momentum or energy equations, 3) other forms of moment equations or equations integrated over only part of the domain. Since it is hoped to carry the chosen profiles to the point where  $h$  intersects the axis and there is strong uncertainty concerning  $\mu_T$  behavior pointwise, it is considered dangerous in turbulent flow to use centerline equations. An additional velocity moment of the momentum equation is also considered dangerous because it would heavily weight the high-speed outer portion of the flow field. This objection can also be raised against a moment of the energy equation, but a stronger objection is the algebraic complexity introduced. A suggestion made by Peters<sup>7</sup> has therefore been followed. The momentum equation is integrated from 0 to  $r = h + \ell/2$ . This yields

$$\begin{aligned}
 d/dx (\rho_e u_e^2 A_4) - u' d/dx (\rho_e u_e A_5) &= \rho_e u_e r'^2/2 du_e/dx \\
 &+ r' \mu_T \partial u / \partial r \Big|_{r'} \\
 A_4 &= \int_0^{r'} \rho u^2 / e u_e^2 r dr, & A_5 &= \int_0^{r'} \rho u / \rho_e u_e r dr \quad (12)
 \end{aligned}$$

where primed quantities indicate values evaluated at  $r = h + \ell/2$ . This equation is reasonably simple algebraically and there is some confidence that  $\mu_T$  may be evaluated properly in the high shear region of the main shear layer.

The turbulent viscosity is taken from Schetz<sup>8</sup> and modified into a form used by Alber with the necessary constant being chosen to match the eddy viscosity to known results for an incompressible free shear layer under similarity conditions.<sup>9</sup> The result is

$$\mu_T = 0.04649 \rho_e u_e / a (A_2 - \rho_c u_c / \rho_e u_e A_u) \quad (13)$$

where  $a$  is the base radius. This form should be adequate until the rear stagnation point is reached, at which point the term involving  $A_u$  would be removed and  $\mu_T \propto A_2$ , the momentum area, in accord with Alber's work.

The differential equations of Eqs. (7-10) and (12), together with the profiles of Eq. (11), may be manipulated to the following system of first-order nonlinear differential equations:

$$a_{ij} dy_j/dx = b_i$$

$$y_1 = \lambda \quad y_2 = h \quad y_3 = P \quad y_4 = P_H \quad y_5 = u_e \quad (14)$$

Guessing a base pressure and value of  $P_H$ , these equations may be integrated downstream from the base if the initial shear layer thickness  $\lambda$  is known. The other initial conditions are  $P = 1/2$ ,  $h = a$ , and  $u_e$ , determined from the guessed base pressure. When  $h$  intersects the axis, this approach is terminated for a more realistic set of profiles to carry the computation through the stagnation point<sup>6</sup> and critical point<sup>2</sup> singularities.

### Results

For checkout purposes it was first decided to assume a constant pressure exterior flow with the additional restriction that  $du_e/dx = 0$ . This allows the dropping of one of the differential equations since  $du_e/dx$  is now known. Furthermore, the approximate method of characteristics may be dropped. Computations are shown for a case in Fig. 1. The assumed value of the stagnation enthalpy in the core flow at the base was equal to the freestream value, and other values are shown in the figure with  $p_b$  the assumed value of the base pressure. Indeed the solution proceeds smoothly until the shear layer lower edge intersects the axis. Notable is the extremely high value of the reversed flow velocity attained,  $u_c \approx 2360$  fps. In fact, this is locally supersonic with respect to the local speed of sound, which gives an indication of trouble for the over-all method. The chosen base pressure ratio is close to that experimentally known for this case.<sup>10</sup> Consequently, this



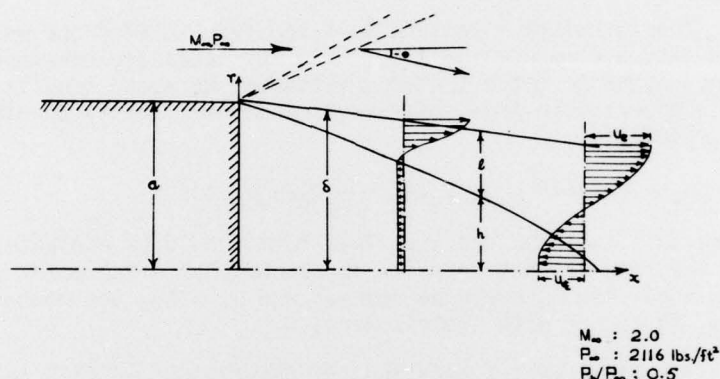


Fig. 1 Scale drawing of near wake flow assuming a base pressure and integrating downstream of the base. Constant pressure case.

case was not expected to yield a character far from the actual solution, at least for a short distance downstream.

To see the effects of interaction of the viscous flow with the external flow, the full set of equations was integrated together with the approximate method of characteristics. Under several different forms of the fifth equation, including centerline equations, the solution would start as shown in Fig. 1, but a saddle point singularity was encountered before the shear layer intersected the axis. This also occurred under several different assumptions for the exterior flow, including Prandtl-Meyer flow. Although there appears no good argument that a critical point should not appear until after the rear stagnation point, it is believed that, since other solutions<sup>2,6</sup> have never encountered an upstream singularity, these simplistic profiles are at fault. Furthermore, the appearance of a supersonic reversed flow in the constant pressure case is not considered tenable. Although the profiles used appear adequate for integration, say one base radius downstream, it appears that, when one nears the complex reattachment region, they are inadequate. Consequently, current efforts are being directed to conversion to a different profile type before the shear layer intersects the axis.

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- <sup>10</sup>Reid, J. and Hastings, R. C., "Experiments on the Axisymmetric Flow over After-Bodies and Basis at  $M=2.0$ ," Rep. Aero. 2628, 1959, Royal Aircraft Establishment, Farnborough, England.

#### Discussion

OHRENBERGER (TRW Systems, Redondo Beach, California):

I could not agree more with your comments in regard to experience with these difficulties. What you are going through right now is essentially what we went through for about a year and a half of trying all kinds of things. There are many things that we learned regarding very similar problems that you are encountering. This is not the time or place to go through

those. One thing that I might say in passing is that we did find that the use of the centerline momentum equation did not work for the very simple reason that the integral method requires that the pressure vary axially but not radially. In the laminar case, especially near the base, large radial pressure gradients truly do exist, and we found that applying the mean axial pressure gradient to the axis flow simply did not work at all. In fact our solution actually behaved similarly to what you experience with this singular behavior before you get to the stagnation point.

STRAHLE:

I appreciate those comments and I understand. There is an additional difficulty in the turbulent case about using any centerline equation at all, and that is I do not know what the turbulent exchange coefficient is on the centerline. That is why I wanted to use an integral equation, but then one does not pick up a singularity and so I think I am stuck.

OHRENBERGER:

It is interesting that you have tried a moment equation in lieu of centerline energy equation and you did not experience in that case a singularity akin to the thermal singularity that we experienced.

STRAHLE:

No, we went right through and you cannot see it in the equations either. The beauty of the centerline equations is that you can see it.

SEDNEY (Ballistics Research Laboratories, Aberdeen Proving Ground, Maryland):

I wanted to ask about the characteristics. You recall Chapman's work<sup>1</sup> many years ago. For the axisymmetric case, he had to stop before he got to the axis and so all his results for that calculation have a sting.

STRAHLE:

I do not understand the difficulty because the method of characteristics never comes down to the axis. We only use it in the inviscid portion of the flow and that is off the axis.

<sup>1</sup>Chapman, D., NACA Rep. 1051, 1951.



SEDNEY:

Yes, but he found he had to stop it at a significant distance off the axis, otherwise the calculation would blow up.

STRAHLE:

It has never really gotten to that small of  $r$  yet. The viscous portion fills a good portion of the axis region compared to the base height, down to about a half at a minimum, and we have not experienced any trouble with the method of characteristics. We define the boundary layer thickness of the viscous zone thickness, and we use the boundary layer equations between  $r$  equals zero and that particular thickness. That becomes an unknown in the problem, but it is only at that exterior edge of the viscous layer that we apply the method of characteristics.

TANG (McDonnell Douglas Astronautics, Huntington Beach, California):

I think you have employed a two-dimensional velocity profile for the axisymmetric case, is that true?

STRAHLE:

That is true. It is a simple profile, it has some of the characteristics of the flows that were needed, and I did not see anything that prevented us from using it in an axisymmetric case. Of course, the values of the profile parameters will be different when we march downstream in the axially symmetric case compared with the two-dimensional case.

KOOKER (Ballistics Research Laboratories, Aberdeen Proving Ground, Maryland):

I realize that in the example case you have discussed, the heat release is assumed to be zero. However, the general case as shown in your initial slide might contain a substantial heat release in the fuel entrainment region. Since your equations do not account for radial pressure gradients, is it possible that the integral approach may encounter additional difficulties due to a lack of communication in the radial direction?

STRAHLE:

When we go to the exterior part and actually include external burning, I am planning to continue to use an integral



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technique but including the effect of the radial pressure gradient. Now that may be too ambitious a statement, but I am not there yet. I am still trying to get to the rear stagnation point.

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